3. Logaritmische grafieken en exponentiële verbanden.

Opgave 3.1 **Aflezen van coördinaten in een enkellog-grafiek.**

De coördinaten van de punten A, B, C, D en E.

A: $(2; 10^{1,1}) \rightarrow (2; 12{,}6)$

B: $(3; 10^{1,8}) \rightarrow (3; 63)$

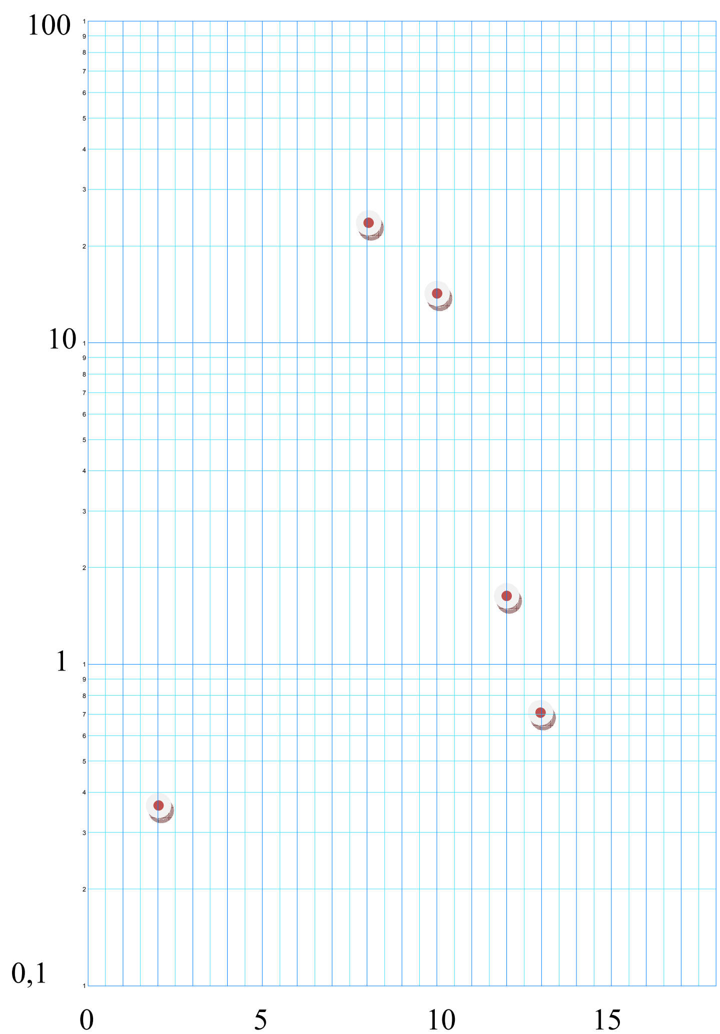
C: $(5{,}3; 10^{2,1}) \rightarrow (5{,}3; 126)$

D: $(4; 10^{2.5}) \rightarrow (4; 316)$

E: $(6{,}9; 10^{2,62}) \rightarrow (6{,}9; 417)$

Opgave 3.2 **Uitzetten van punten in een enkellog-grafiek.**

Stel de juist schaalverdeling in en via copy en paste krijg de volgende grafiek.



Opgave 3.3 **Aflezen van coördinaten in een dubbellog-grafiek.**

De coördinaten van de punten A, B, C, D en E.

A: $(10^{1,2}; 10^{-0,4}) \rightarrow (16; 0,4)$

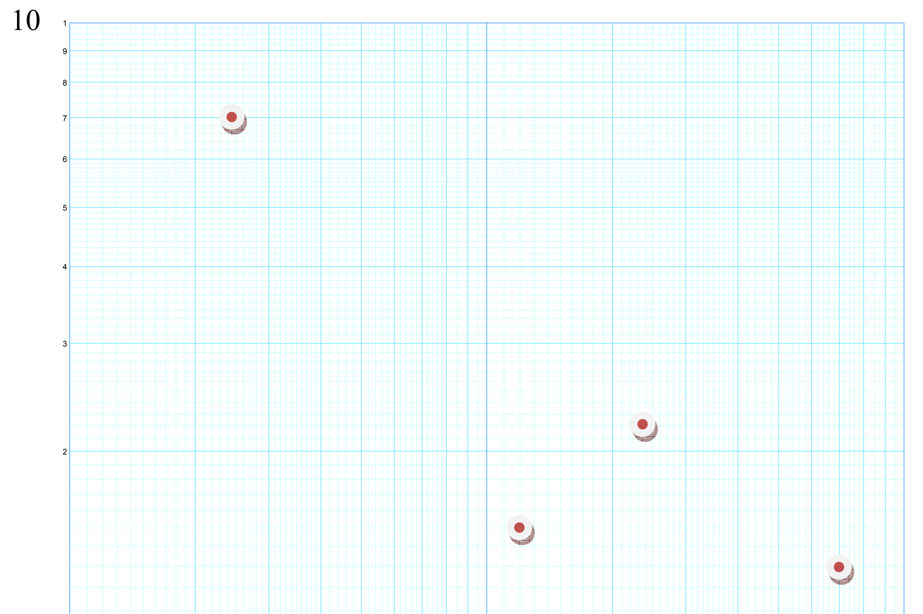
B: $(10^{0,55}; 10^{0,1}) \rightarrow (3,5; 1,3)$

C: $(10^{0,2}; 10^{0,6}) \rightarrow (1,6; 4,0)$

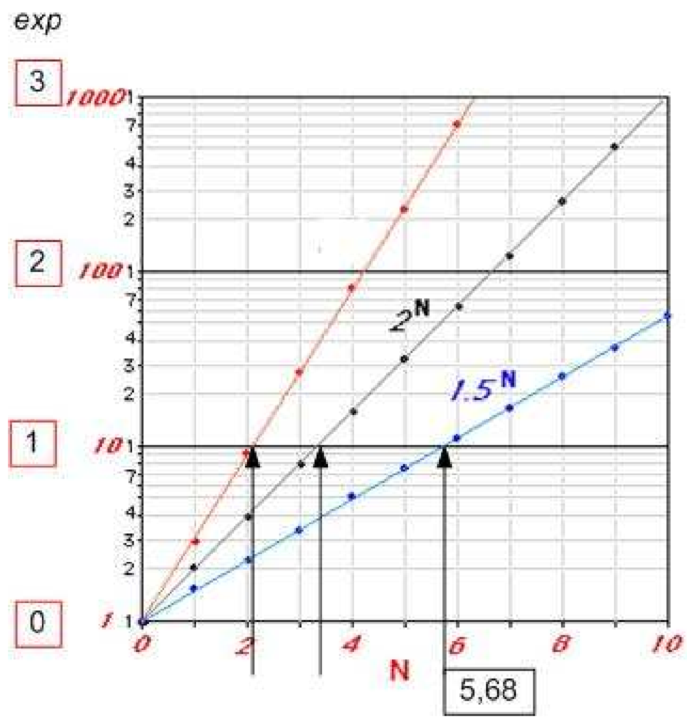
D: $(10^{1,13}; 10^{0,22}) \rightarrow (13; 1,7)$

E: $(10^{1,65}; 10^{0,88}) \rightarrow (45; 7,6)$

Opgave 3.4 **Uitzetten van punten in een dubbellog-grafiek.**



Opgave 3.5 **Grondtal bepalen met enkellog-grafiek.**



a $2^N = (10^{0,3})^N = 10^{0,3N}$

b $helling\ rood = \dfrac{1}{2,1} = 0,48$

exponent $= 0,48 \cdot N$

c $(10^{0,48})^N = 3^N$

Opgave 3.6 **Groei van je spaarrekening.**

a $K(1) = 1,065 \times 1780 = 1896$

b $K(10) = (1,065)^{10} \times 1780 = 1,877 \times 1780 = 3341$

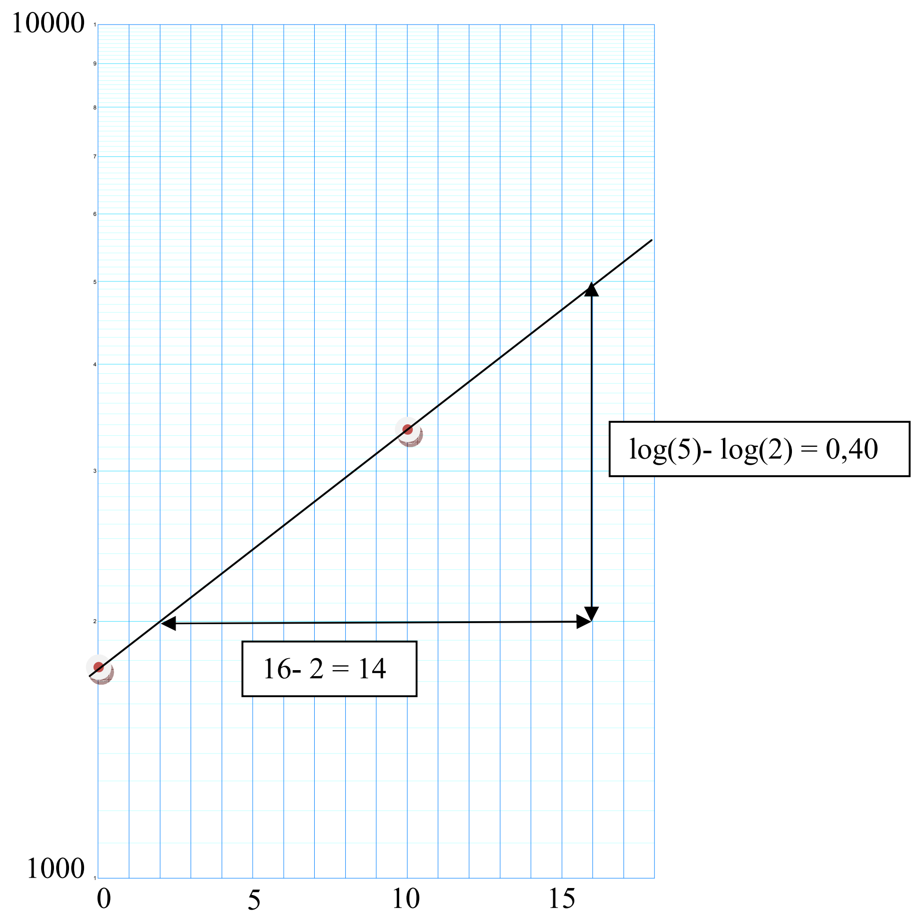
c $rente\ na\ 10\ jaar = K(10) - K(0) = 3341 - 1780 = 1561$

d zie grafiek hierna

e $(1,065)^N = 10^{0,0273 \cdot N}$

f $helling = \dfrac{0,4}{14} = 0,028$

klopt met antwoord g



Opgave 3.7 **Oefenen met $^{a}\log(b)$.**

Bereken en controleer.

a voorbeeld:

$$^{2}\log(8) = \frac{^{10}\log(8)}{^{10}\log(2)} = 3 \quad \text{controle}: 2^3 = 8$$

b $$^{1,04}\log(1,6) = \frac{^{10}\log(1,6)}{^{10}\log(1,04)} = 12 \quad \text{controle}: 1,04^{12} = 1,60$$

c $$^{3}\log(100) = \frac{^{10}\log(100)}{^{10}\log(3)} = 4,19 \quad \text{controle}: 3^{4,19} = 99,8$$

d $$^{3}\log(9) = \frac{\log(9)}{\log(3)} = 2 \qquad \text{controle}: 3^2 = 9$$

e $$^{3}\log(3^{11}) = 11 \quad \textit{want} \quad 3^{11} = 3^{11}$$

Opgave 3.8 **Bereken de waarde van *x*.**

Bereken en controleer.

a voorbeeld:

$$5x = {}^{3}\log(5) \rightarrow x = \frac{{}^{3}\log(5)}{5} = \frac{\log(5)}{5\log(3)} = 0{,}293$$

controle : $5 \times 0{,}293 = {}^{3}\log(5)$ klopt

b $1200 = 1000 \cdot (1{,}06)^{x} \rightarrow 1{,}06^{x} = \frac{1200}{1000} \rightarrow 1{,}06^{x} = 1{,}2$

$\rightarrow x = {}^{1{,}06}\log(1{,}2)$ *exact*

$\rightarrow x = \frac{\log(1{,}2)}{\log(1{,}06)} = 3{,}13$ *afgerond*

controle : $1000 \cdot (1{,}06)^{3{,}13} = 1200{,}1$ klopt

c $2^{x} = 1000 \rightarrow x = {}^{2}\log(1000)$ *exact*

$\rightarrow x = \frac{\log(1000)}{\log(2)} = 9{,}97$ *afgerond*

controle : $2^{9{,}97} = 1003$ klopt

d $3 \cdot 4^{1{,}5x} = 500 \rightarrow 4^{1{,}5x} = \frac{500}{3} = 166{,}7$

$\rightarrow 1{,}5x = {}^{4}\log(166{,}7) = \frac{\log(166{,}7)}{\log(4)} = 3{,}69 \rightarrow x = \frac{3{,}69}{1{,}5} = 2{,}46$

controle : $3 \cdot 4^{3{,}69} = 499{,}7$ klopt

e $x^{2} = 1000 \rightarrow x = 1000^{1/2}$ *of* $x = \sqrt{1000}$ *exact*

$\rightarrow x = 31{,}6$ *afgerond*

controle : $31{,}6^{2} = 998{,}6$ klopt

f $x^{1{,}5} = 1000 \rightarrow x = 1000^{1/1{,}5} = 1000^{2/3} = 100$

controle : $100^{1{,}5} = 1000$ klopt

g $5^{x} = 100 \rightarrow x = {}^{5}\log(100)$ *exact*

$\rightarrow x = \frac{\log(100)}{\log(5)} = 2{,}86$ *afgerond*

controle : $5^{2{,}86} = 99{,}8$ klopt

h ${}^{x}\log(1000) = 3 \rightarrow x^{3} = 1000 \rightarrow x = 10$

i ${}^{5}\log(2x) = 0{,}1 \rightarrow 5^{0{,}1} = 2x \rightarrow x = \frac{5^{0{,}1}}{2} = 0{,}587$

controle : ${}^{5}\log(2 \times 0{,}587) = 9{,}97 \cdot 10^{-2}$ *klopt*

j $x^3 = 100 \rightarrow x = 100^{1/3}$ *of* $x = \sqrt[3]{100}$ *exact*

$\rightarrow x = 4{,}64$ *afgerond*

controle : $4{,}64^3 = 99{,}9$ klopt

k $5 \cdot {}^2\log(x) = 10 \rightarrow {}^2\log(x) = 2 \rightarrow x = 2^2 = 4$

controle : $5 \cdot {}^2\log(4) = 5 \times 2 = 10$

l ${}^2\log(x^5) = 10 \rightarrow x^5 = 2^{10} \rightarrow x = 2^2 = 4$

m $x = 2{,}5^{-0{,}2} \rightarrow x = 0{,}833$

Opgave 3.9 **Groei van je spaarrekening en berekening met logaritme.**

a $K(10) = K(0) \cdot 1{,}04^{10} \rightarrow K(10) = 1780 \times 1{,}04^{10} = 2635$

b $K(n) = K(0) \cdot 1{,}04^n \rightarrow 1{,}7 \cdot K(0) = K(0) \times 1{,}04^n$

$$\rightarrow 1{,}04^n = 1{,}7 \rightarrow n = {}^{1{,}04}\log(1{,}7) = \frac{\log(1{,}7)}{\log(1{,}04)} = 13{,}5$$

na 13,5 jaar is je kapitaal met 70% gegroeid

c $K(n) = K(0) \cdot 1{,}04^n \rightarrow 3000 = 1780 \times 1{,}04^n$

$$\rightarrow 1{,}04^n = \frac{3000}{1780} = 1{,}685 \rightarrow n = {}^{1{,}04}\log(1{,}685) = \frac{\log(1{,}685)}{\log(1{,}04)} = 13{,}3$$

na 13,3 jaar is het kapitaal 3000

d $K = 1780 + 300 = 2080$

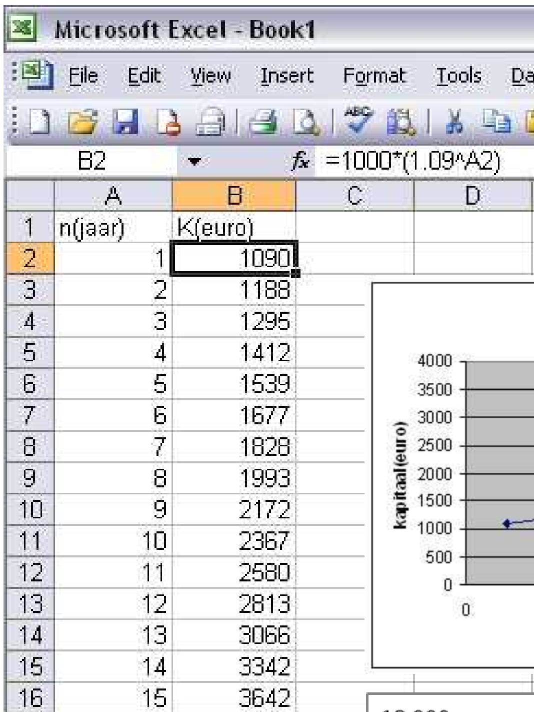
$K(n) = K(0) \cdot 1{,}04^n \rightarrow 2080 = 1780 \times 1{,}04^n$

$$\rightarrow 1{,}04^n = \frac{2080}{1780} = 1{,}169 \rightarrow n = {}^{1{,}04}\log(1{,}169) = \frac{\log(1{,}169)}{\log(1{,}04)} = 4{,}0$$

na 4,0 jaar is de rente 300

Opgave 3.10 **Groei van je spaarrekening en Excel.**

$$K(0) = 1000$$
$$groeifactor = 1{,}09$$
$$\rightarrow K(n) = 1000 \cdot (1{,}09)^n$$

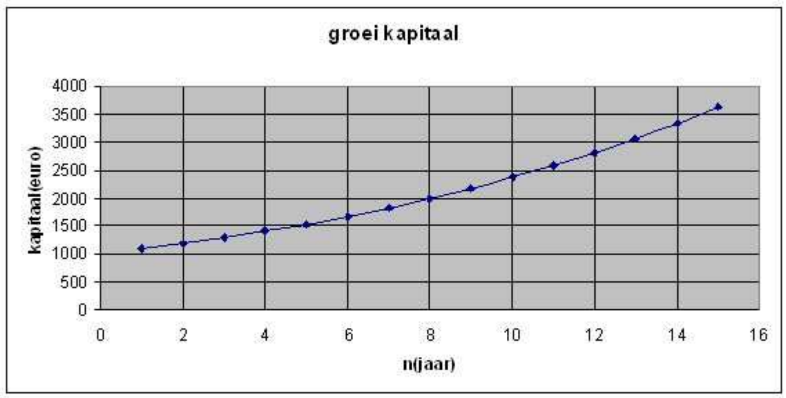


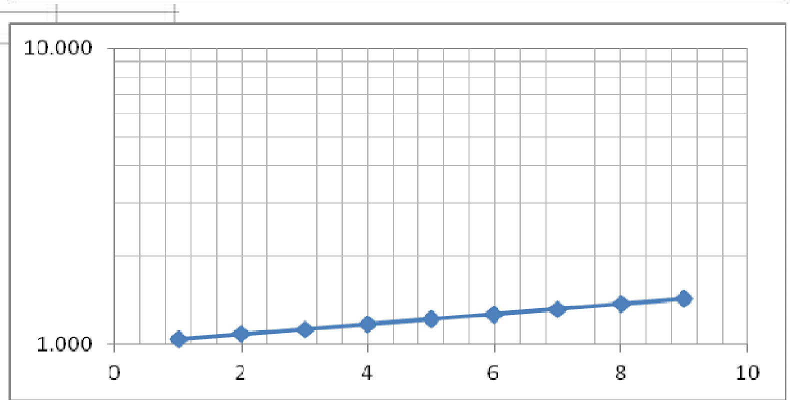
Microsoft Excel - Book1

File Edit View Insert Format Tools Da

B2 =1000*(1.09^A2)

	A	B	C	D
1	n(jaar)	K(euro)		
2	1	1090		
3	2	1188		
4	3	1295		
5	4	1412		
6	5	1539		
7	6	1677		
8	7	1828		
9	8	1993		
10	9	2172		
11	10	2367		
12	11	2580		
13	12	2813		
14	13	3066		
15	14	3342		
16	15	3642		





Opgave 3.11 **Exponent bepalen met dubbellogaritmisch papier.**

a $x = 5 \rightarrow y = 2 \times 5^3 = 250 \quad \textit{klopt met grafiek}$

$x = 5 \rightarrow \log(x) = \log(5) = 0{,}7 \quad \textit{klopt}$

$x = 5 \rightarrow \log(y) = \log(250) = 2{,}4 \quad \textit{klopt}$

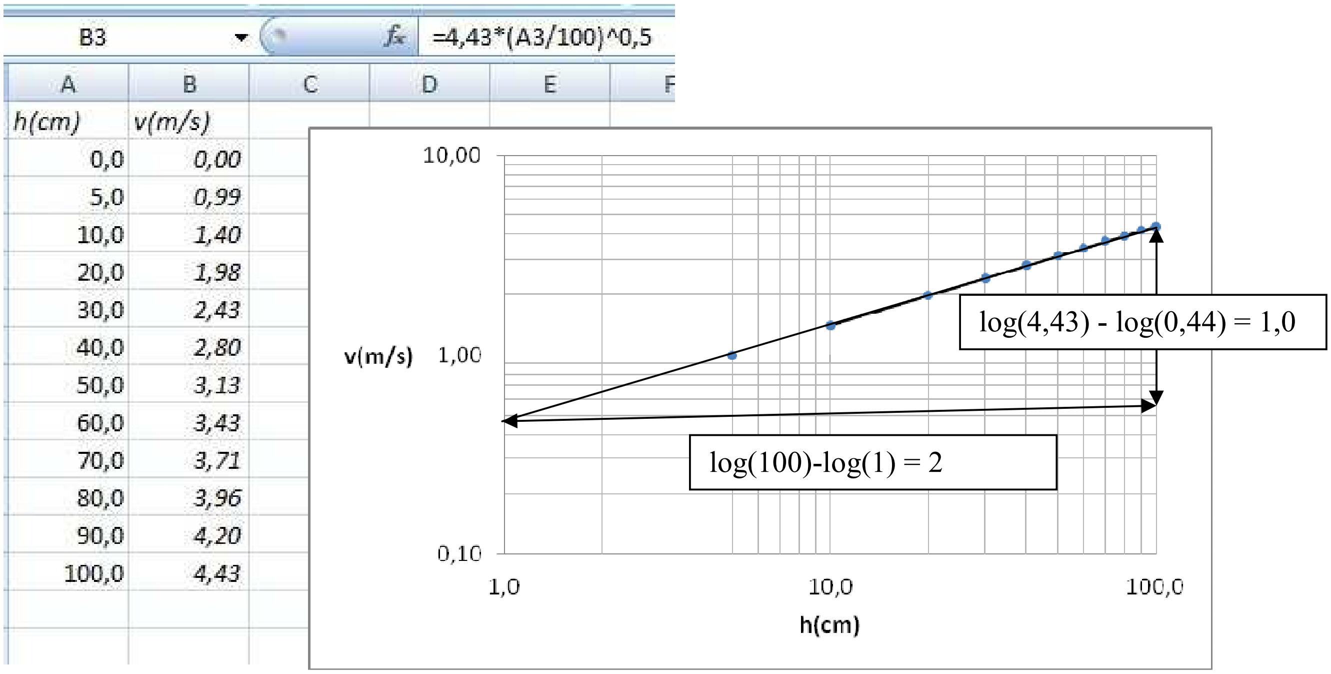
b $\log(x) = 0{,}5 \rightarrow x = 10^{0{,}5} = 3{,}16 \quad \textit{klopt met grafiek}$

$\log(x) = 0{,}5 \rightarrow x = 3{,}16 \rightarrow y = 2 \times 3{,}16^3 = 63 \quad \textit{klopt}$

$\log(x) = 0{,}5 \rightarrow \log(y) = \log(63) = 1{,}8 \quad \textit{klopt}$

Opgave 3.12 Exponent bepalen in de formule voor de uitstroomsnelheid.

a



B3 =4,43*(A3/100)^0,5

A	B
h(cm)	v(m/s)
0,0	0,00
5,0	0,99
10,0	1,40
20,0	1,98
30,0	2,43
40,0	2,80
50,0	3,13
60,0	3,43
70,0	3,71
80,0	3,96
90,0	4,20
100,0	4,43

b $v = 4{,}43 \cdot h^{1/2}$ dan ook $\log(v) = \log(4{,}43 \cdot h^{1/2})$

ofwel ${}^{10}\log(v) = {}^{10}\log(p) + q\,{}^{10}\log(h)$

$p = 4{,}43$ *en* $q = 0{,}5$

c $helling = \dfrac{1}{2}$ klopt met formule

Opgave 3.13 Radioactief verval.

a $T_{1/2}({}^{60}Co) = 5 \text{ jaar} \rightarrow n = \dfrac{12 \text{ jaar}}{5 \text{ jaar}} = 2{,}4$

$N = N(0) \cdot (\frac{1}{2})^n \rightarrow N(12) = 100\% \times (0{,}5)^{2{,}4} = 18{,}9\ \%$

b $m(0) = 1200 \text{ g} \rightarrow m(12) = 18{,}9\ \%\ van\ 1200 \text{ g}$

$\rightarrow \text{m}(12) = 0{,}189 \times 1200 = 227 \text{ g}$

c $N(3) = 100\% \times (0{,}5)^3 = 12{,}5\ \%$

d $N = 5\% \rightarrow 5\% = 100\% \times (0{,}5)^n$ *links en rechts delen door* 100

$\rightarrow 0{,}05 = (0{,}5)^n \rightarrow n = {}^{0,5}\log(0{,}05) = \dfrac{\log(0{,}05)}{\log(0{,}5)} = 4{,}32$

$\rightarrow t = 4{,}32 \times T_{1/2} = 4{,}32 \times 5 \text{ jaar} = 21{,}6 \text{ jaar}$

Opgave 3.14 **De leeftijd van een fossiel bepalen met behulp van radioactieve kernen.**

a $N = N(0)\cdot(\frac{1}{2})^n \rightarrow 5\% = 100\% \times (0{,}5)^n$

$\rightarrow 0{,}05 = (0{,}5)^n \rightarrow n = {}^{0,5}\log(0{,}05) = \dfrac{\log(0{,}05)}{\log(0{,}5)} = 4{,}32$

$\rightarrow t = 4{,}32 \times T_{1/2}({}^{14}C) \rightarrow t = 4{,}32 \times 5730 \text{ jaar} = 24840 \text{ jaar}$

afgerond : $t = 25000$ jaar

b $t = 1200 \text{ jaar} \rightarrow n = \dfrac{1200}{5730} = 0{,}209$

$N = N(0)\cdot(\frac{1}{2})^n \rightarrow N = 100\% \times (0{,}5)^{0,209} = 86{,}5\ \%$

Het percentage is 87% van het percentage in levende natuur.

c $t = 10000 \text{ jaar} \rightarrow n = \dfrac{10000}{5730} = 1{,}745$

$N = N(0)\cdot(\frac{1}{2})^n \rightarrow N = 100\% \times (0{,}5)^{1,745} = 29{,}8\ \%$

Er is nog 29,8 % van de ${}^{14}C$ - atomen over.

Opgave 3.15 **Herleiden formule voor radioactief verval.**

a

$N = 100\cdot(2)^{-t/20} = 100\cdot(0{,}5)^{t/20}$

$N = 100\cdot(0{,}5)^{t/20}$ (N in % *en* t in dagen)

$n = t/20 \rightarrow T_{1/2} = 20$ dagen

b $2 = 4^{0,5}$

$N = 100\cdot(2)^{-t/20} = 100\cdot(4)^{-0,5t/20} = 100\cdot(4)^{-t/40}$

c met formule vraag b) : $N = 100\cdot(4)^{-20/40} = 50\ \%$

met formule vraag a) : $N = 100\cdot(2)^{-20/20} = 50\%$

beide formules geven hetzelfde antwoord

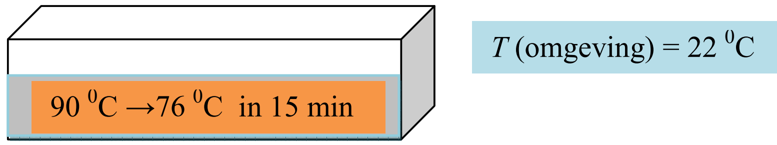
d $N = 100\cdot(4)^{-t/40} = 100\cdot(\frac{1}{4})^{+t/40} = 100\cdot(0{,}25)^{t/40} = 100\cdot(0{,}25)^n$

met $n = t/40$

n is het aantal keren dat de beginwaarde $4\times$ zo klein geworden is.

Opgave 3.16 **Berekening aan afkoelproces.**

a



$$\Delta T(0) = 90 - 22 = 68\ ^0C \quad en \quad \Delta T(15\text{ min}) = 76 - 22 = 54\ ^0C$$

$$\Delta T = \Delta T(0)\cdot(0{,}5)^n \rightarrow 54 = 68\cdot(0{,}5)^n \rightarrow (0{,}5)^n = \frac{54}{68} = 0{,}794$$

$$\rightarrow n = {}^{0,5}\log(0{,}794) = \frac{\log(0{,}794)}{\log(0{,}5)} = 0{,}333$$

$$n = \frac{t}{T_{1/2}} \rightarrow T_{1/2} = \frac{t}{n} \rightarrow T_{1/2} = \frac{15\text{ min}}{0{,}333} = 45\text{ min}$$

b

$$n = \frac{30\text{ min}}{45\text{ min}} = 0{,}667$$

$$\Delta T = \Delta T(0)\cdot(0{,}5)^n \rightarrow \Delta T = 68\cdot(0{,}5)^{0{,}667} \rightarrow \Delta T = 42{,}8\ ^0\mathrm{C}$$

$$\rightarrow T(bak) = \Delta T + 22^0C = 64{,}8\ ^0C \quad afgerond: T = 65\ ^0C$$

$$temperatuurdaling = 90 - 65 = 25\ ^0C$$

c

$$\Delta T = 8\ ^0C$$

$$\Delta T = \Delta T(0)\cdot(0{,}5)^n \rightarrow 8 = 68\cdot(0{,}5)^n \rightarrow (0{,}5)^n = \frac{8}{68} = 0{,}118$$

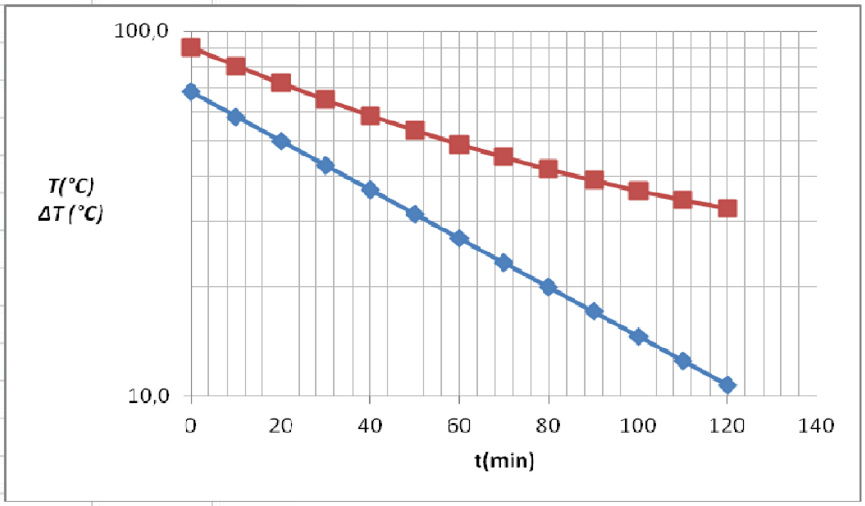
$$\rightarrow n = {}^{0,5}\log(0{,}118) = \frac{\log(0{,}118)}{\log(0{,}5)} = 3{,}08$$

$$\rightarrow t = n\times T_{1/2} = 3{,}08\times 45\text{ min} = 139\text{ min} = 2\text{ u en }19\text{ min}$$

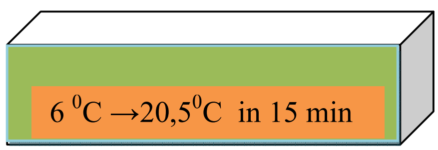
d De waardes van T en ΔT zijn in een enkellog-diagram uitgezet tegen t. De grafiek van ΔT loopt lineair,
Je kunt het Excel-bestand opgave2.41.xlsx downloaden van de site www.vervoortboeken.nl

B2 f_x =68*(0,5)^(A2/45)

A	B	C
t (min)	ΔT (°C)	T (°C)
0	68,0	90,0
10	58,3	80,3
20	50,0	72,0
30	42,8	64,8
40	36,7	58,7
50	31,5	53,5
60	27,0	49,0
70	23,1	45,1
80	19,8	41,8
90	17,0	39,0
100	14,6	36,6
110	12,5	34,5
120	10,7	32,7



Opgave 3.17 **Berekening aan opwarmproces.**



T (omgeving) = 20,5 ^{0}C

a $\Delta T(0) = 14{,}5\ {}^0C$

volgens grafiek : $T_{1/2} = 2{,}3$ min

b $t = 3 \text{ min} \rightarrow n = \dfrac{3}{2{,}3} = 1{,}30$

$\Delta T = \Delta T(0)\cdot(0{,}5)^n \rightarrow \Delta T(3 \text{ min}) = 14{,}5\cdot(0{,}5)^{1,3} = 5{,}9\ {}^0\text{C}$

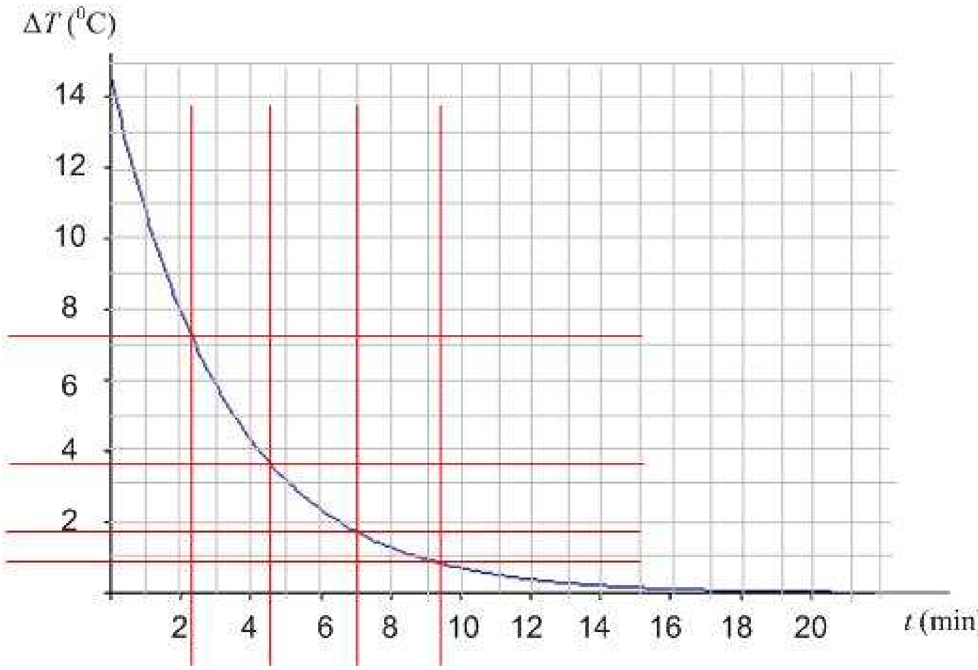
$\rightarrow T = 20{,}5 - \Delta T \rightarrow T = 20{,}5 - 5{,}9 = 14{,}6\ {}^0C$

c $T = 18\ {}^0C \rightarrow \Delta T = 20{,}5 - 18 = 2{,}5\ {}^0C$

$\Delta T = \Delta T(0)\cdot(0{,}5)^n \rightarrow 2{,}5 = 14{,}5\cdot(0{,}5)^n \rightarrow (0{,}5)^n = \dfrac{2{,}5}{14{,}5} = 0{,}172$

$\rightarrow n = {}^{0,5}\log(0{,}172) = \dfrac{\log(0{,}172)}{\log(0{,}5)} = 2{,}54$

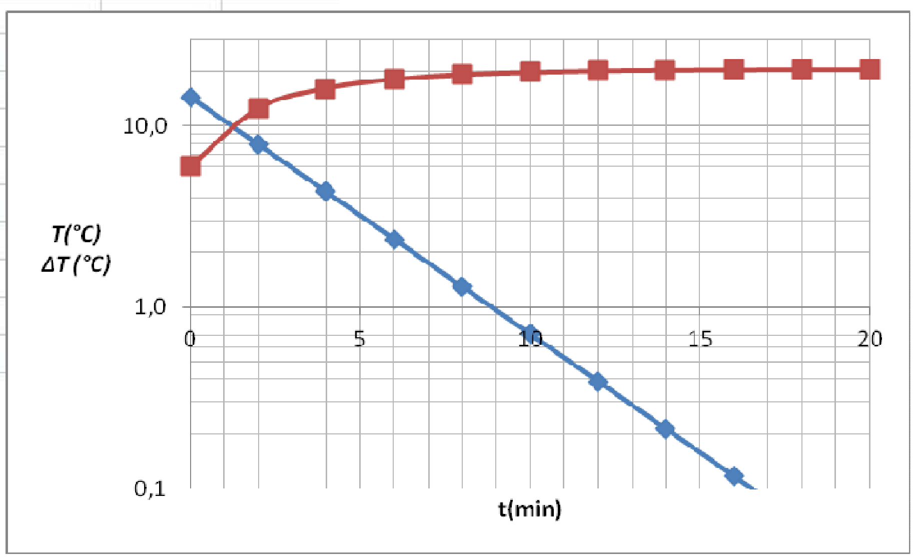
$\rightarrow t = n \times T_{1/2} \rightarrow t = 2{,}54 \times 2{,}3 = 5{,}8$ min



d De waardes van T en ΔT zijn in een enkellog-diagram uitgezet tegen t. De grafiek van ΔT loopt lineair,
Je kunt het Excel-bestand opgave2.42.xlsx downloaden van de site www.vervoortboeken.nl

B2 =14,5*(0,5)^(A2/2,3)

A	B	C
t (min)	ΔT (°C)	T (°C)
0	14,5	6,0
2	7,9	12,6
4	4,3	16,2
6	2,4	18,1
8	1,3	19,2
10	0,7	19,8
12	0,4	20,1
14	0,2	20,3
16	0,1	20,4
18	0,1	20,4
20	0,0	20,5



Opgave 3.18 **Berekening aan absorptie van röntgenstraling.**

a $E_d = E_i \cdot (0{,}5)^n \rightarrow E_d = 100\% \cdot (0{,}5)^3 = 12{,}5\%$

b $n = \dfrac{d}{d_{1/2}} = 7$

$E_d = E_i \cdot (0{,}5)^n \rightarrow E_d = 100\% \cdot (0{,}5)^7 = 0{,}78\%$

c $E_d = E_i \cdot (0{,}5)^n \rightarrow E_d = 100\% \cdot (0{,}5)^{4{,}6} = 4{,}1\%$

d $E_d = 0{,}8 \cdot E_i$ = 80% van invallende energie

e $E_d = E_i \cdot (0{,}5)^n \rightarrow 9{,}5\% = 100\% \cdot (0{,}5)^n \rightarrow 0{,}5^n = 0{,}095$

$\rightarrow n = {}^{0{,}5}\log(0{,}095) = \dfrac{\log(0{,}095)}{\log(0{,}5)} = 3{,}4$

f $\rightarrow d = n \times d_{1/2} = 3{,}4 \times d_{1/2}$

Opgave 3.19 **Berekeningen aan doordringdiepte van straling.**

a $n = \dfrac{x}{d_{1/2}} \rightarrow n = \dfrac{6}{9} = 0{,}667$

$I(x) = I_0 \cdot (0{,}5)^n \rightarrow I(6\text{ mm}) = 90 \cdot (0{,}5)^{0{,}667} = 57\ W/m^2$

b

$I(x) = I_0 \cdot (0{,}5)^n \rightarrow 30 = 90 \cdot (0{,}5)^n \rightarrow (0{,}5)^n = \dfrac{30}{90} = 0{,}333$

$\rightarrow n = {}^{0{,}5}\log(0{,}333) = \dfrac{\log(0{,}333)}{\log(0{,}5)} = 1{,}586$

$\rightarrow d = n \times d_{1/2}\text{ mm} = 1{,}586 \times 9\text{ mm} = 14\text{ mm}$

c $I(x) = I_0 \cdot (0{,}5)^n \rightarrow 25\% = 100\% \cdot (0{,}5)^n \rightarrow (0{,}5)^n = 0{,}25$

$\rightarrow n = {}^{0{,}5}\log(0{,}25) = \dfrac{\log(0{,}25)}{\log(0{,}5)} = 2$

$\rightarrow d = n \times d_{1/2} \rightarrow d = 2 \times 9\text{ mm} = 18\text{ mm}$

d $I(x) = I_0 \cdot (0{,}5)^n \rightarrow 20\% = 100\% \cdot (0{,}5)^n \rightarrow (0{,}5)^n = 0{,}20$

$\rightarrow n = {}^{0{,}5}\log(0{,}20) = \dfrac{\log(0{,}20)}{\log(0{,}5)} = 2{,}32$

$d_{1/2}(\text{hout}) = 22{,}4\text{ cm}$

$\rightarrow d = n \times d_{1/2} \rightarrow d = 2{,}32 \times 22{,}4\text{ cm} = 52\text{ cm}$

Opgave 3.20 **Berekeningen aan bacteriegroei 1.**

a $N = N(0) \cdot 2^n \rightarrow N = 1 \times 2^{10} = 2^{10}$

1 bacterie is gegroeid tot 2^{10} bacterien

b $T_2(Eschericia) = 17\,\text{min}$

$N = 2^8 \rightarrow t = 8 \times T_2 \rightarrow t = 8 \times 17\text{ min} = 136\text{ min}$

c $N = N(0) \cdot 2^n \rightarrow 1000 = 2^n$

$\rightarrow n = {}^2\log(1000) = \dfrac{\log(1000)}{\log(2)} = 9{,}97$

$t = 9{,}97 \times 17 = 169\text{ min}$

d In de aanpassingsfase: $c = 10^1\ \text{bact}/\text{mL}$

e In de logfase is c toegenomen van $10^1\ \text{bact}/\text{mL}$ tot $10^9\ \text{bact}/\text{mL}$

Dus $c = \dfrac{10^9}{10^1} = 10^8 \times$ zo groot

Opgave 3.21 **Berekeningen aan bacteriegroei 2.**

a $c = 10 \cdot 2^{20}\ \text{bact}/\text{mL} = 1{,}05 \cdot 10^7\ \text{bact}/\text{mL}$

b $N = 10.000 \times 10 = 10^5\ \text{bact}/\text{mL}$

c volgens grafiek is hiervoor 16 – 8 = 8 uur nodig.

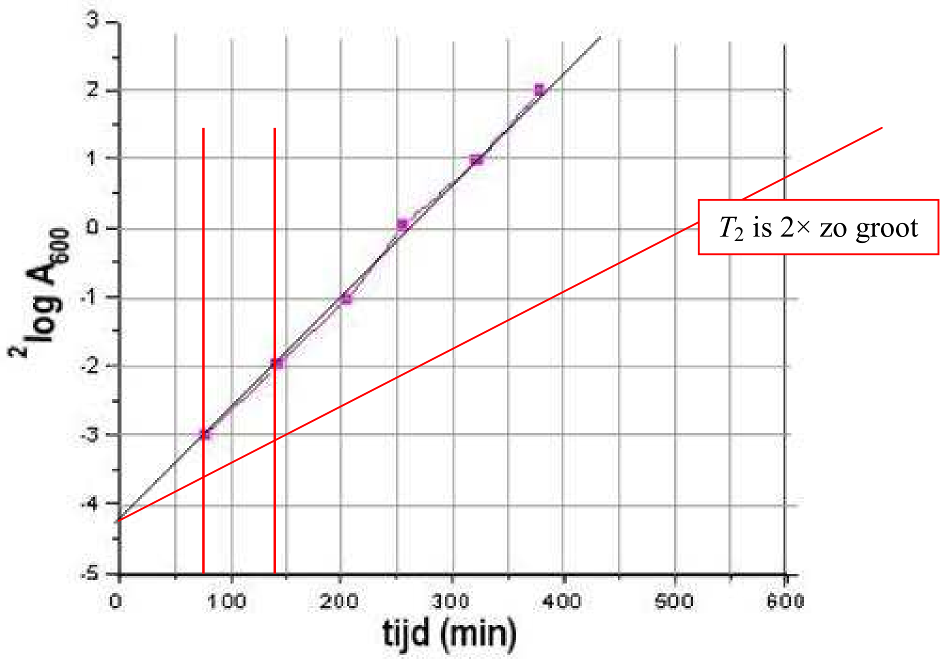
d $N = N(0) \cdot 2^n \rightarrow 10^5 = 10^1 \cdot 2^n \rightarrow 2^n = 10^4$

$\rightarrow n = {}^2\log(10^4) = \dfrac{\log(10^4)}{\log(2)} = 16{,}6$

$n = 16{,}6$ delingen in 8 uur

e $T_2 = \dfrac{8 \times 60\text{ min}}{16{,}6\text{ delingen}} = 29\text{ min}$

Opgave 3.22 **Gebruik van een andere logschaal.**



a $^2\log(A_{600}) = -3 \rightarrow A_{600} = 2^{-3} = 0{,}125$

b afstand tussen rode lijnen in grafiek

c $^2\log(A_{600}) = -4{,}2 \rightarrow A_{600} = 2^{-4{,}2} = 0{,}054$

d De afstand tussen rode lijnen wordt 2 × zo groot.
De helling wordt dus 2 × zo klein.