

6 Goniometrische functies

Opgave 6.1
tool 6.8



Rekenen met de goniometrische verhoudingsgetallen.

a $2\pi \text{ rad} = 360^0 \rightarrow \alpha = 1 \text{ rad} = \frac{360^0}{2\pi} = 57,3^0$

b $\sin(1) = 0,841$
 $\cos(1) = 0,540$ *Stel de juiste eenheid in op je ZRM*
 $\tan(1) = 1,56$

c $\sin(2) = 0,909$
 $\cos(2) = -0,416$
 $\tan(2) = -2,19$

d $\sin(4) = -0,757$
 $\cos(4) = -0,654$
 $\tan(4) = 1,16$

e $\sin(5) = -0,959$
 $\cos(5) = 0,284$
 $\tan(5) = -3,38$

Opgave 6.3

tool 6.9



Ontbinden van een vector.

a $|a| = 25 \quad \theta = 0,32 \text{ rad}$
 $a_x = |a| \cdot \cos(\theta) \rightarrow a_x = 25 \cdot \cos(0,32) = 23,7$
 $R_y = |R| \cdot \sin(\theta) \rightarrow R_y = 25 \cdot \sin(0,32) = 7,86$

b $a_x = 10 \quad a_y = 16$
 $\tan(\theta) = \frac{a_y}{a_x} \rightarrow \tan(\theta) = \frac{16}{10} = 1,6 \rightarrow \theta = \arctan(1,6) = 1,01 \text{ rad}$
 $a = \sqrt{a_x^2 + a_y^2} = \sqrt{10^2 + 16^2} = 18,9$

c $a_x = 10 \quad \theta = 0,20 \text{ rad} \rightarrow |a| = \frac{a_x}{\cos(\theta)} \rightarrow |a| = \frac{10}{\cos(0,2)} = 10,2$

d $a_y = 12 \quad |a| = 20$
 $a_x^2 + a_y^2 = a^2 \rightarrow a_x^2 = 400 - 144 = 256 \rightarrow a_x = 16$
 $\tan(\theta) = \frac{a_y}{a_x} \rightarrow \tan(\theta) = \frac{12}{16} = 0,75 \rightarrow \theta = \arctan(0,75) = 0,643 \text{ rad}$

e

$$a_y = 10 \rightarrow \theta = 1,20 \text{ rad}$$

$$a_y = |a| \cdot \sin(\theta) \rightarrow |a| = \frac{a_y}{\sin(\theta)} = \frac{10}{\sin(1,20)} = 10,73$$

$$a_x^2 = a^2 - a_y^2 \rightarrow a_x^2 = 10,73^2 - 10^2 = 15,11 \rightarrow a_x = \sqrt{15,11} = 3,89$$

oefenen 6.1

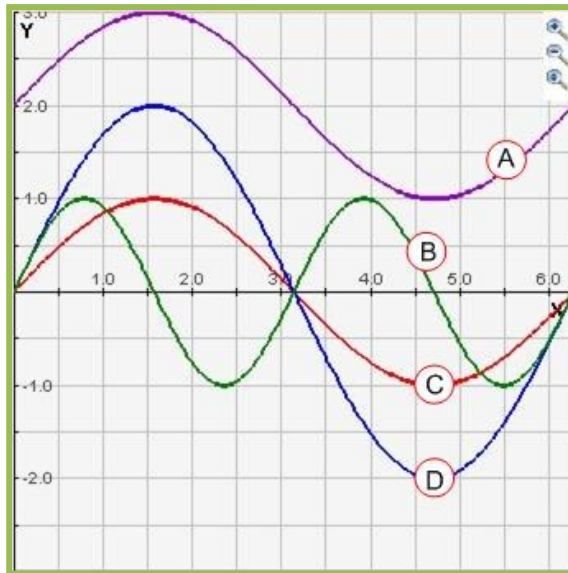


Meer oefening en/of uitleg nodig?
Site van Herman Hofstede

Opgave 6.5

Onderzoek goniometrische functies.

desmostool



A: $k(x) = \sin(x) + 2$
amplitude = 1 1 periode op $[0, 2\pi]$ grafiek is 2 naar boven verschoven

Als x verandert van 0 tot 2π verandert de hoek α van 0 tot 2π rad.

B: $h(x) = \sin(2x)$
amplitude = 1 2 periodes op $[0, 2\pi]$

Als x verandert van 0 tot 2π verandert de hoek α van 0 tot 4π rad ($2 \times$ rond).

C: $f(x) = \sin(x)$
amplitude = 1 1 periode op $[0, 2\pi]$

D: $g(x) = 2 \cdot \sin(x)$
amplitude = 2 1 periode op $[0, 2\pi]$

Opgave 6.7

Verschuiven, versterken en kortere periode.

desmostool

A: $k(x) = 2 \sin(2x)$
amplitude = 2 2 periodes op $[0, 2\pi]$

B: $h(x) = \sin(x - \pi/2)$
amplitude = 1 1 periode op $[0, 2\pi]$ $\pi/2$ naar rechts verschoven

De beginhoek $\alpha_0 = -\pi/2$ of $3\pi/2$

C: $g(x) = \sin(x + \pi/2)$
amplitude = 1 1 periode op $[0, 2\pi]$ $\pi/2$ naar links verschoven

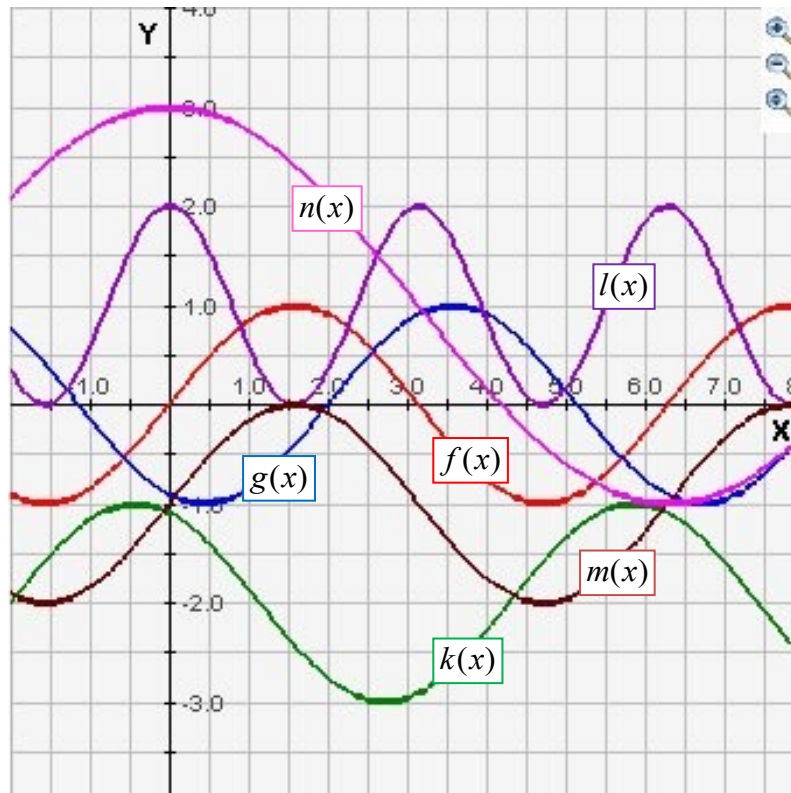
De beginhoek $\alpha_0 = \pi/2$

D: $f(x) = \sin(x)$
amplitude = 1 1 periode op $[0, 2\pi]$

Opgave 6.9

Schetsen van de grafiek $y = A \sin(kx + c) + b$

desmostool



Opgave 6.11

De functie $u(t) = A \sin\left(\frac{2\pi}{T}t + \alpha_0\right)$ bij een veer-massa systeem.

tool 6.18



a
$$T = 2\pi\sqrt{\frac{m}{C}} \rightarrow T^2 = 4\pi^2 \cdot \frac{m}{C} \rightarrow C = \frac{4\pi^2 m}{T^2}$$

$$\rightarrow C = \frac{4\pi^2 \times 0,25}{1^2} = 9,87 \text{ N/m}$$

b
$$u(t) = A \sin\left(\frac{2\pi}{T}t + \alpha_0\right) \rightarrow u(t) = 0,2 \sin\left(\frac{2\pi}{1} \cdot t + 0\right)$$

$$\rightarrow u(t) = 0,2 \sin(2\pi \cdot t)$$

c
$$T = 2\pi\sqrt{\frac{m}{C}} \rightarrow T = 2\pi\sqrt{\frac{0,1}{9,87}} = 0,632 \text{ s}$$

$$u(t) = 0,2 \sin\left(\frac{2\pi}{0,632} \cdot t\right) \rightarrow u(t) = 0,2 \sin(9,94 \cdot t)$$

d
$$u(t) = 0,2 \sin(9,94 \cdot t + 3,14)$$

e Voor de beweging van een massa aan een veer geldt:

$$u(t) = 30 \sin(1,44t + \pi/2)$$

$$\frac{2\pi}{T} = 1,44 \rightarrow T = \frac{2\pi}{1,44} = 4,36 \text{ s}$$

$$T^2 = 4\pi^2 \cdot \frac{m}{C} \rightarrow m = \frac{C \cdot T^2}{4\pi^2} = \frac{9,87 \times 4,36^2}{4 \times \pi^2} = 4,75 \text{ kg}$$

$$\alpha_0 = \pi/2 \text{ rad}$$

Opgave 6.13

De functie $u(t) = A \sin\left(\frac{2\pi}{T}t + \alpha_0\right)$ bij een slinger.

tool 6.21



a
$$T = 2\pi \cdot \sqrt{\frac{l}{g}} \rightarrow T = 2\pi \cdot \sqrt{\frac{0,80}{9,81}} = 1,79 \text{ s}$$

b
$$u(t) = 10 \cos\left(\frac{2\pi}{1,79} \cdot t + 0\right) \rightarrow u(t) = 10 \cos(3,51 \cdot t)$$

c
$$T = 2\pi \cdot \sqrt{\frac{l}{g}} \rightarrow T = 2\pi \cdot \sqrt{\frac{1}{9,81}} = 2,01 \text{ s}$$

$$u(t) = 10 \cos\left(\frac{2\pi}{2,01} \cdot t + 0\right) \rightarrow u(t) = 10 \cos(3,14 \cdot t)$$

tool 6.22



d
$$u(t) = 0,1 \sin(3,14 \cdot t + \pi)$$

e

$$u(t) = 9 \cos\left(2t + \frac{\pi}{2}\right)$$

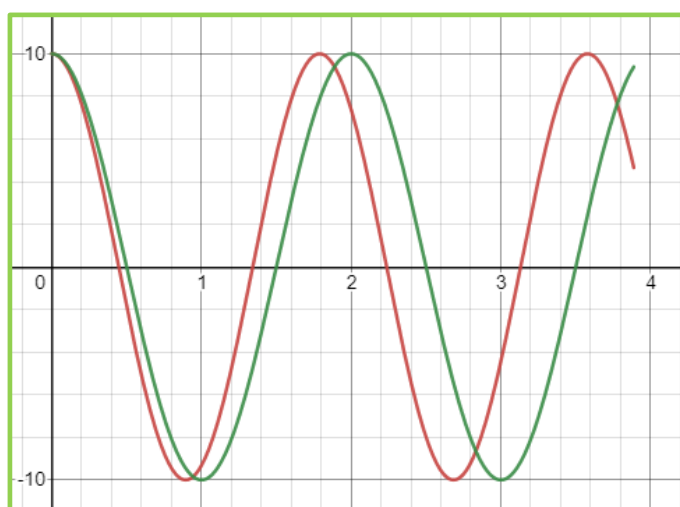
$$\frac{2\pi}{T} = 2 \rightarrow T = \frac{2\pi}{2} = 3,14 \text{ s}$$

$$T = 2\pi \cdot \sqrt{\frac{l}{g}} \rightarrow T^2 = 4\pi^2 \cdot \frac{l}{g} \rightarrow l = \frac{T^2 \cdot g}{4\pi^2}$$

$$\rightarrow l = \frac{3,14^2 \times 9,81}{4\pi^2} = 2,45 \text{ m}$$

$$\alpha_0 = \frac{\pi}{2}$$

f



Opgave 6.15

tool 6.24



tool 6.25



Lopende golf

voor punt A: $u(t) = \sin(4\pi \cdot t)$

$$\lambda = AE = 32 \text{ cm} = 3,2 \text{ cm}$$

b

$$\lambda = v \cdot T \rightarrow v = \frac{\lambda}{T}$$

$$\frac{2\pi}{T} = 4\pi \rightarrow T = \frac{2\pi}{4\pi} = 0,5 \text{ s}$$

$$v = \frac{3,2 \text{ cm}}{0,5 \text{ s}} = 6,4 \text{ cm/s}$$

c

$$u_A(t) = \sin(4\pi \cdot t)$$

$$u_B(t) = \sin\left(4\pi \cdot t - \frac{\pi}{2}\right)$$

d

$$u_A(t) = \sin(4\pi \cdot t)$$

$$u_C(t) = \sin(4\pi \cdot t - \pi)$$

e $u_A(t) = \sin(4\pi \cdot t)$
 $u_D(t) = \sin(4\pi \cdot t - \frac{3}{2}\pi)$

f tool 6.22

Opgave 6.17
tool 6.28



Spanning van de wandcontactdoos.

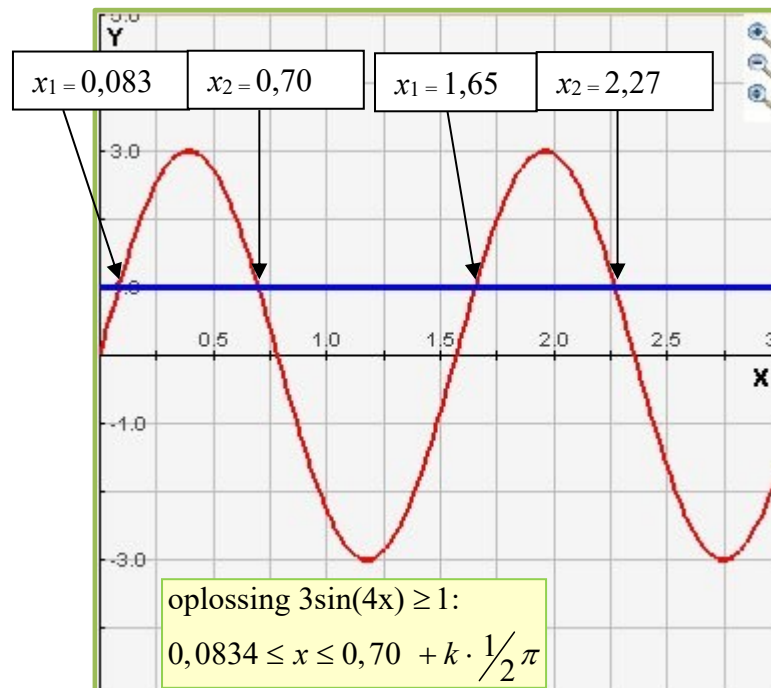
a $U(t) = 325 \sin(\frac{2\pi}{0,02} \cdot t) \rightarrow U(t) = 325 \sin(100\pi \cdot t)$

b $U(t) = |325 \cdot \sin(100\pi t)|$

Opgave 6.19
desmostool

Enkele goniometrische vergelijkingen.

a $3 \sin(4x) \geq 1$
 $3 \sin(4x) = 1 \rightarrow \sin(4x) = \frac{1}{3} \quad \arcsin(\frac{1}{3}) = 0,340 \text{ rad}$
 $\sin(4x_1) = \sin(0,340) \rightarrow 4x_1 = 0,34 + k \cdot 2\pi \rightarrow x_1 = 0,0834 \pm k \cdot \frac{1}{2}\pi$
 $x_1 = 0,0834; 1,65 + k \cdot \frac{1}{2}\pi \quad k \in \mathbb{Z}$
en
 $\sin(4x_2) = \sin(\pi - 0,34) \rightarrow 4x_2 = 2,80 \rightarrow x_2 = 0,70 + k \cdot \frac{1}{2}\pi$
 $x_2 = 0,70; 2,27 + k \cdot \frac{1}{2}\pi$



b $4 \cos(x - 2) = 1$

$$\cos(x - 2) = 0,25 \quad \arccos(0,25) = 1,32$$

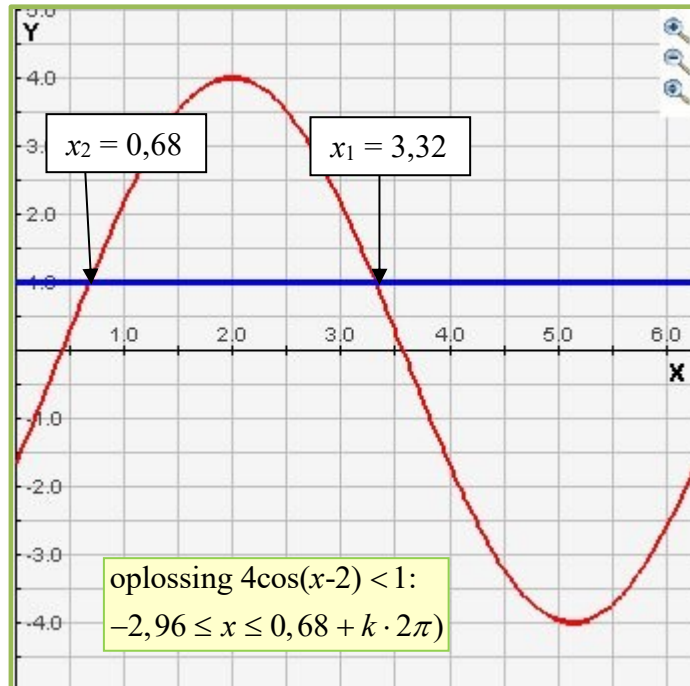
$$\cos(x_1 - 2) = \cos(1,32) \rightarrow x_1 - 2 = 1,32 \pm k \cdot 2\pi$$

$$\rightarrow x_1 = 3,32 + k \cdot 2\pi \quad k \in \mathbb{Z}$$

en

$$\cos(x_2 - 2) = \cos(2\pi - 1,32) \rightarrow x_2 - 2 = 2\pi - 1,32 + k \cdot 2\pi$$

$$\rightarrow x_2 = 0,68 + k \cdot 2\pi$$



c $2 \sin(2x - \frac{\pi}{4}) \geq 0,75$

$$2 \sin(2x - \frac{\pi}{4}) = 0,75 \rightarrow \sin(2x - \frac{\pi}{4}) = 0,375 \quad \arcsin(0,375) = 0,384$$

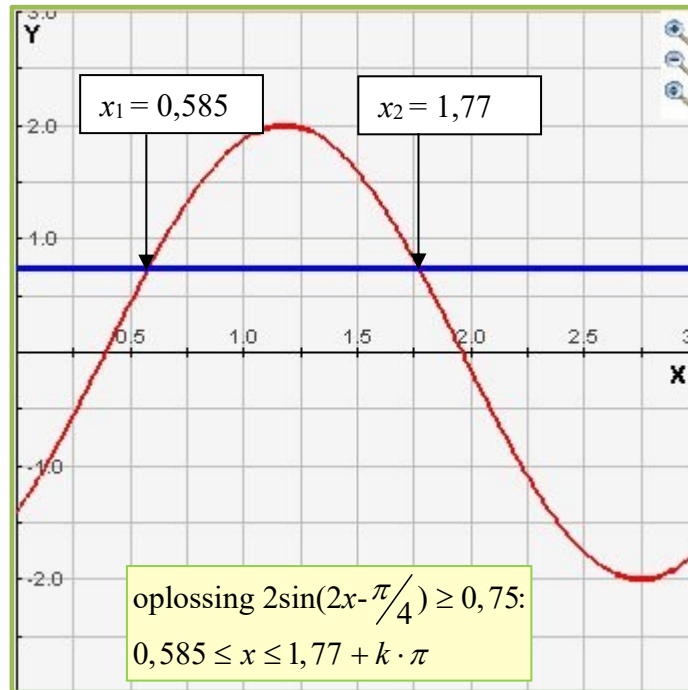
$$\sin(2x_1 - \frac{\pi}{4}) = \sin(0,384) \rightarrow 2x_1 - \frac{\pi}{4} = 0,384 + k \cdot 2\pi$$

$$\rightarrow 2x_1 = 1,17 + k \cdot 2\pi \rightarrow x_1 = 0,585 + k \cdot \pi \quad k \in \mathbb{Z}$$

en

$$\sin(2x_2 - \frac{\pi}{4}) = \sin(\pi - 0,384) \rightarrow 2x_2 - \frac{\pi}{4} = 2,76 + k \cdot 2\pi$$

$$\rightarrow 2x_2 = 3,54 + k \cdot 2\pi \rightarrow x_2 = 1,77 + k \cdot \pi$$



d

$$0,5 \tan(0,5x - \frac{\pi}{3}) = 2$$

$$0,5 \tan(0,5x - \frac{\pi}{3}) = 2 \rightarrow \tan(0,5x - \frac{\pi}{3}) = 4 \quad \arctan(4) = 1,33$$

$$\tan(0,5x_1 - \frac{\pi}{3}) = \tan(1,33) \rightarrow 0,5x_1 - \frac{\pi}{3} = 1,33 + k \cdot 2\pi$$

$$\rightarrow 0,5x_1 = 2,38 + k \cdot 2\pi \rightarrow x_1 = 4,76 + k \cdot 4\pi \quad k \in \mathbb{Z}$$

en

$$\tan(0,5x_2 - \frac{\pi}{3}) = \tan(\pi + 1,33) \rightarrow 0,5x_2 - \frac{\pi}{3} = 4,47 + k \cdot 2\pi$$

$$\rightarrow 0,5x_2 = 5,52 + k \cdot 2\pi \rightarrow x_2 = 11,0 + k \cdot 4\pi$$

$$0,5 \tan(0,5x - \frac{\pi}{3}) = 2 \text{ als } x = 4,47 \vee x = 11,0 + k \cdot 4\pi$$

